Cylindrical Post with Gap in a Rectangular Waveguide

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Abstract—**The problem of a thin hollow post, of the type often used for semiconductor mounting structures, in a rectangular waveguide is considered for the case where the post has a variable gap located at some position on it. The solution for the reflection coefficient in the waveguide for the case where there is only one propagating mode in the waveguide. A comparison of theoretical results and experimental measurements of the reflection coefficient, as a function of gap size, shows the theory yields a high degree of accuracy.**

I. Introduction

The PROBLEM of an inductive metal post in a rectangular waveguide has been widely studied and it is well known that discontinuities of this form may be used to realise a variety of waveguide subsystems. Variable length posts are often employed as the adjustable elements in matching networks, filters, and so on. Tabulated experimental results for this situation have long be available [1]. A more recent theoretical analysis of the variable length post has been performed by Williamson [2] which uses a similar method as that used for the coaxially driven probe problem [3].

The problem considered here is that of a hollow cylindrical post in a rectangular waveguide for the case where the post has a variable gap located at some position on it. It is assumed that this is is an obstacle problem where there is an incident field in the waveguide. This situation occurs frequently in microwave systems as a mounting structure for many microwave semiconductor devices [4], [5]. The analysis of the problem considered here uses the method used by Keam [6] for the coaxially driven post with gap, along with that used by Williamson for the variable length post. A brief comparison of theoretical and experimental results is also given.

II. Theory

Consider the case where the hollow post shown in Fig. 1 is located in a rectangular waveguide of cross sectional dimensions d and h. The post has a radius $r = a$ and is assumed to be hollow. The situation considered here is that were the post extends the full width across the waveguide and has a gap of width $2g$ which is positioned at a height of $z = z_s$, where z_s is the height from the base of one end of the post to the middle of the gap. It is assumed that the frequency of operation is such that the TE_{10} mode is the only propagating mode in the waveguide.

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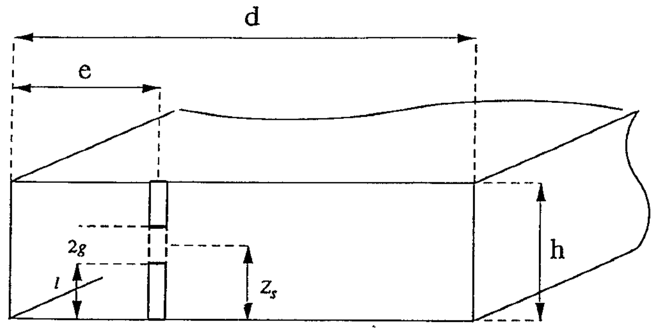

Fig. 1. Sectional view of a variable gap hollow, cylindrical post in a rectangular waveguide.

In terms of the (x, y, z) co-ordinate system in the waveguide, the electric field may be written as an incident field, a reflected field back from the post, and a transmitted field past the post, ie. $E_z^{\text{inc}} = E_i \sin(\pi x/d)e^{-jk_{10}y}, E_z^{\text{ref}} = E_r \sin(\pi x/d)e^{jk_{10}y}$, and $E_z^{\text{trans}} = E_t \sin(\pi x/d)e^{-jk_{10}y}$. Where k_{10} is the propagation wave number for the TE_{10} mode, ie. $k_{10} = k\sqrt{1-(\pi/kd)^2}$

The analysis here is concerned with thin posts for which the current induced by the incident electric field is axially symmetric on the post surface. For this case the field scattered by the induced current is the same in both the reflected and transmitted direction, and so, $E_z^{\text{trans}} = E_z^{\text{inc}} + E_z^{\text{ref}}$.

Using the same approach adopted in [3], [6] for the coaxially driven problem, an integral equation may be formulated which relates the electric field in the gap to the current which would be induced if no gap were present and the current Green's function due to a delta function source located on the post. Thus no assumptions need be made about the form of the current distribution induced on the post. It is straightforward to show [7] that this integral equation is

$$I_{\text{ind}}(z) = \int_{z_s-g}^{z_s+g} E_z(a,z')I^\delta(z,z')\,dz' \qquad \text{for} \quad z_s - g < z < z_s + g \tag{1}$$

where

$$I^\delta(z,z') = \sum_{m=0}^{\infty} B_m \cos\frac{m\pi z}{h}\cos\frac{m\pi z'}{h} \tag{2}$$

where the B_m coefficients are defined below. The unknown electric field in the gap may be expressed as [2]

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$$E_z(a,z) = \left\{E_i h \sin\frac{\pi e}{d} J_0 o(ka)\right\} \frac{1/\pi}{\sqrt{g^2 - z^{*2}}} \cdot \sum_{p=0}^{\infty} e_p T_p(z^*/g) \quad (3)$$

where $z^* = z - z_s$. T_p is a pth order Chebyshev polynomial of the first kind, J_0 is a Bessel function of the first kind, and e_p are the unknown coefficients to be found. The formulation of (3) uses the result that the current induced on a post with no gap is [3]

$$I_{\text{ind}}(z) = E_i h \sin\frac{\pi e}{d} J_0(ka) B_0 \quad (4)$$

and so upon substituting (4) and (3) into (1) it may be shown that

$$B_0\delta_0 = \sum_{p=0}^{\infty} e_p \sum_{m=0}^{\infty} B_m J_s(m\gamma) J_p(m\gamma) C_{s,m} C_{p,m} \quad s = 0, 1, 2, \cdots \quad (5)$$

where J_s is an s order Bessel function of the first kind, $C_{p,m}$ is given by

$$C_{p,m} = \begin{cases} (-1)^{p/2}\cos m\theta & \text{for } p \text{ even} \\ (-1)^{(p+1)/2}\sin m\theta & \text{for } p \text{ odd} \end{cases} \quad (6)$$

and where $\gamma = \pi g/h$ and $\theta = \pi z_s/h$. Equation (5) may be solved as a matrix equation [6]. The B_m coefficients are defined as

$$B_m = \frac{j4\pi}{\eta k h q_m^2 I_0(q_m ka) S(q_m ka, q_m kd, e/d)} \quad (7)$$

where $S(q_m ka, q_m kd, e/d)$ is the Williamson array factor for a rectangular waveguide given in [3].

Once the unknown e_p coefficients have been found from (5) it is straightforward to determine the current distribution on the post. It can be shown, using (4), that the total current distribution on the post is given by

$$I_t(z) = E_i h \sin\frac{\pi e}{d} J_0(ka) \left[B_0(1 - e_0) - \sum_{p=0}^{\infty} e_p \sum_{m=0}^{\infty} B_m J_p(m\gamma) C_{p,m} \cos\frac{m\pi z}{h} \right]. \quad (8)$$

Having found the gap electric field $E_z(a,z)$, we can then evaluate the fields and current distribution over the surface of the post. In order to determine the reflection coefficient, R, which is defined as the ratio of E_r to E_i, we need to determine the reflected electric field.

If the frequency is such that only the only the TE_{10} mode can propagate then the expression for the electric field scattered by a current on the post can be shown to be given by

$$E_z(x,y,z) = \frac{-k\eta}{k_{10}d} J_0(ka) I_0 \sin\frac{\pi e}{d} \sin\frac{\pi x}{d} e^{jk_{10}y}. \quad (9)$$

Thus it may be shown by considering (8) and (9) that the reflection coefficient ($R = E_r/E_i$) is given by

$$R = \frac{-4J_0(ka)\sin^2\frac{\pi e}{d}}{k_{10} d S^*(ka, kd, e/d)}(1 - e_0) \quad (10)$$

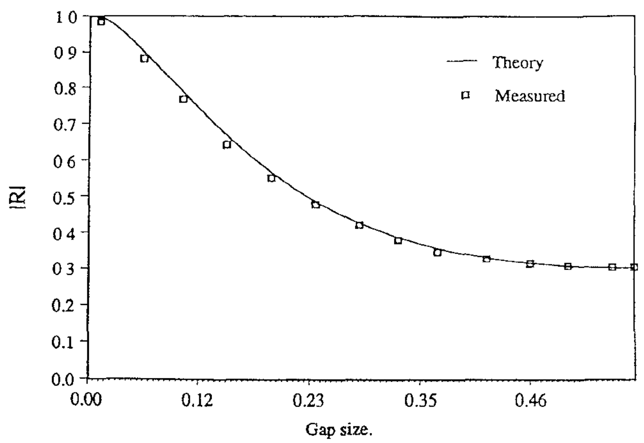

Fig. 2. Theoretical and experimental results for magnitude of reflection coefficient as a function of gap size for the case where $a = 1.525$ mm, $\ell/h = 0.55$, and $f = 4.5$ GHz.

where $S^*(ka, kd, e/d)$ is the rectangular waveguide array factor given in [3]. The transmission coefficient T, is simply given by $T = 1 + R$, and it may be shown from (10) that $|R|^2 + |T|^2 = 1$, as required by the power conservation condition. It is interesting to note that this is of the same form as that derived by Williamson for the variable length post, and that the gap information is contained entirely in the e_0 coefficient. This has the advantage that when considering the design of a semiconductor mounting structure the same equivalent circuit given in [3] may be applied.

III. Experimental Verification

Experimental measurements were performed using a post of radius $a = 1.525$ mm located in the center of a rectangular waveguide of dimensions $d = 47.55$ mm and $h = 22.15$ mm (yielding a single mode operating frequency of approximately 3–6 GHz). The waveguide reflection coefficient of the TE_{10} mode was measured using the HP-8510C network analyzer calibrated[1] to the plane of the post position in the waveguide.

Fig. 2 shows a comparison between theoretical predictions and experimental measurements of the magnitude of the reflection coefficient in the waveguide as a function of the gap size at a frequency of 4.5 GHz. Again the theory is shown to be very accurate even for very small gap sizes. For the case considered here it is clear, at least for small gap sizes, that the magnitude of the reflection coefficient is significantly influenced by the gap size. This may be an important consideration for the design of microwave mounting structures where the size and location of the gap effect the performance of the semiconductor device.

IV. Conclusion

The problem of a thin hollow post in a rectangular waveguide has been considered for the case where the post has a variable gap located at some position on it. The solution

[1] *Transmit-Line-Reflect* calibration.

for the reflection coefficient in the waveguide for the case where there is only one propagating mode was found using the method developed for the coaxially driven gap case, along with the method developed for the variable length hollow post in a waveguide. It was shown the the reflection coefficient, and hence the equivalent circuit, for this case is of the same form as that for the variable length hollow post. This fact may be used to assist in the design of semiconductor mounting structures.

A comparison of theoretical results and experimental measurements of the reflection coefficient as a function of gap size showed the theory yields a high degree of accuracy. It is noted that, for small to moderate gap sizes, the magnitude of the reflection coefficient in the waveguide is significantly affected by the gap size.

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